B-SPLINE CURVE

We have some limitations in Bezier Curve like →

1) The Bezier Curve produced by Bernstein basis fn has limited flexibility.

Numbers of Control points decides the degree of the Polynomial Curve. Ex:- 4 Control points results a Cubic polynomial Curve.

So only one way to reduce the degree of the Curve is to reduce the no. of Controlpoints and vice versa.

2) The Second limitation is that the value of the blending fn is non-zero for all parameter values over the entire Curve.

Due to this change in one vertex, changes the entire Curve and this eliminates the ability to produce a local change with in a Curve.

So B-Spline Curve — Basis-Spline Curve is solution of this limitations of Bezier Curve.

Properties of B-Spline Curve :-

1) B-Spline basis is non-global (LOCAL) effect.
In this each Control point affects the shape of the Curve Only over range of parameter Values where its associated basis fn is non zero.

2) B-spline Curve made up of n+1 Control point

3) B-Spline Curve let us specify the order of basis (k) fn and the degree of the resulting Curve is independent on the no. of vertices.

4) It is possible to change the degree of the resulting Curve with out changing the no. of control points.

5) B-Spline Can be used to define both open & close Curves.

6) Curve generally follows the shape of defining polygon
If we have order $K=4$ then degree will be 3 $P(K)=x^3$

7) The Curve line with in the Convex hull of its defining Polygon.

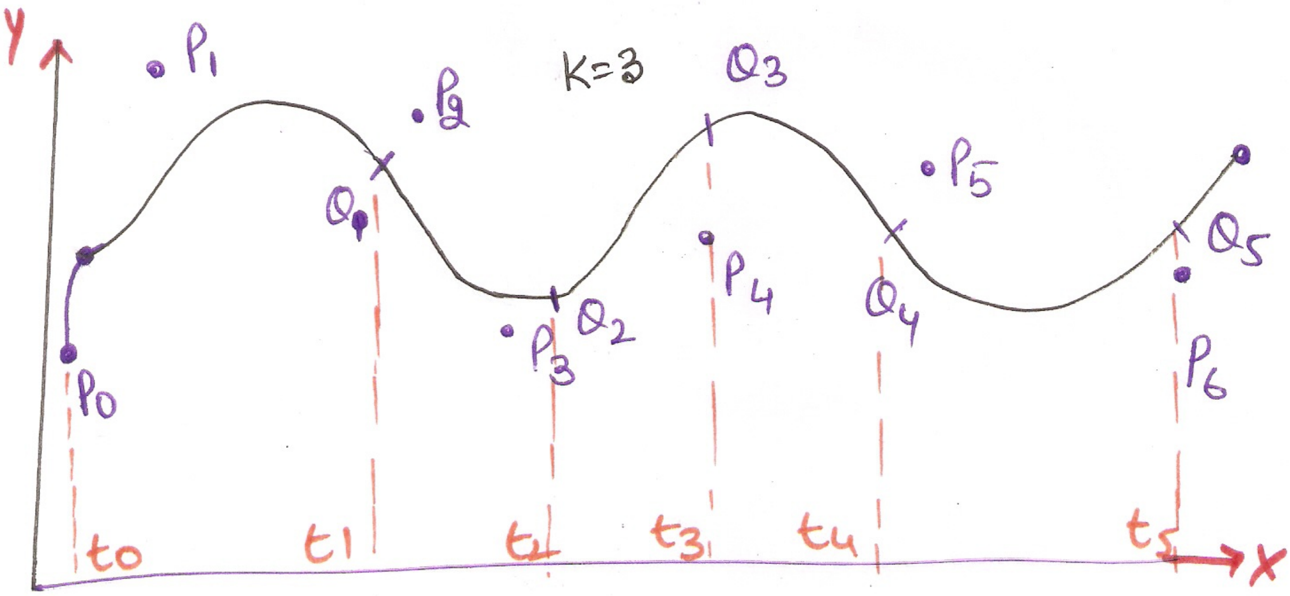
In B-Spline we Segment out the whole Curve which is decided by the order (K). by Formula '$n-K+2$'

for Example:-

If we have 7 control points and order of curve $k=3$ then $n=6$

and this B-Spline curve has segments

$6-3+2=5$



Five Segments Q_1, Q_2, Q_3, Q_4, Q_5

Segment	Control points	Parameter
Q_1	P_0 P_1 P_2	$t_0=0$, $t_1=1$
Q_2	P_1 P_2 P_3	$t_1=1$, $t_2=2$
Q_3	P_2 P_3 P_4	$t_2=2$, $t_3=3$
Q_4	P_3 P_4 P_5	$t_3=3$ $t_4=4$
Q_5	P_4 P_5 P_6	$t_4=4$ $t_5=5$

There will be a join point or knot between Q_{i-1} & Q_i for $i \geq 3$ at the parameter value t_i know as KNOT VALUE [X].

If P(u) be the position vectors along the curve as a fn of the parameter u, a B-Spline curve is given by

$$P(u) = \sum_{i=0}^{n} P_i N_{i,K}(u) \qquad 0 \leq u \leq n-K+2$$

$N_{i,K}(u)$ is B-Spline basis fn

$$N_{i,K}(u) = \frac{(u - X_i) N_{i,K-1}(u)}{X_{i+K-1} - X_i} + \frac{(X_{i+K} - u) N_{i+1,K-1}(u)}{X_{i+K} - X_{i+1}}$$

The values of X_i are the elements of a knot vector satisfying the relation $X_i \leq X_{i+1}$.

The parameter u varies from 0 to n-K+2 along the P(u)

So there are Some Conditions for finding the KNOT VALUES $[x]$

$x_i \ (0 \leq i \leq n+k) \rightarrow$ Knot Values

$x_i = 0$ if $i < k$

$x_i = i-k+1$ if $k \leq i \leq n$

$x_i = n-k+2$ if $i > n$

So as B-Spline Curve has Recursive Equ. so we stop at

$N_{i,k}(u) = 1$ if $x_i \leq u \ x_{i+1}$

$= 0$ otherwise

Example :-

$n=5, \ k=3$

then $x_i \ (0 \leq i \leq 8)$ Knot Values

$x_i \{ \underset{x_0}{0}, \underset{x_1}{0}, \underset{x_2}{0}, \underset{x_3}{1}, 2, 3, 4, 4, \underset{x_8}{4} \}$

$\underbrace{N_{0,3}(u) = (1-u)^2 \cdot N_{2,1}(u)}$

After calculation.

→ when $i=0, \ k=3$ so $i<k$ is true
$x_0 = 0$

→ $i=1, \ k=3 \quad x_1 = 0$

$i=2, \ k=3 \quad x_2 = 0$

$i=3, \ k=3 \quad x_3 = i-k+1 = 3-3+1$
$x_3 = 1$

$i=4, \ k=3 \quad x_4 = i-k+1 \Rightarrow 4-3+1$
$x_4 = 2$

$i=8 \quad k=3 \quad x_8 = i>n \quad n-k+2 = 5-3+2 = 4$
$x_8 = 4$

In this way we will Calculate