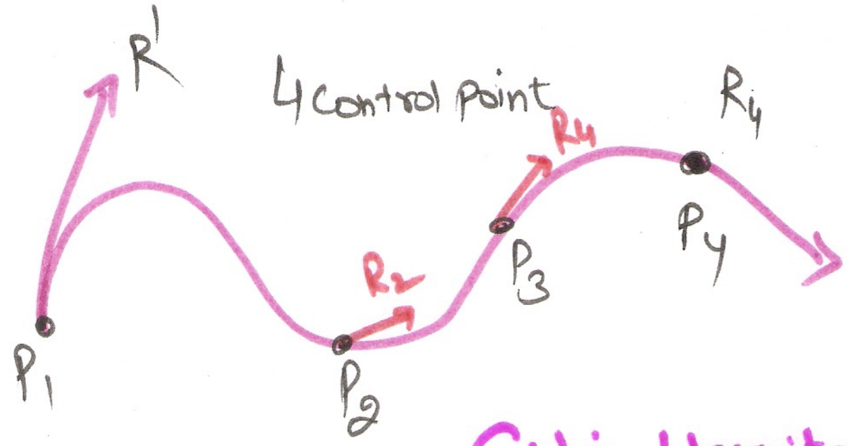
HERMITE SPLINE CURVE

Hermite spline Curve is our Interpolation spline Curve.

The Hermite form of the Cubic polynomial Curve Segment is determined by Constraints on the end points P_1 & P_4 and the Tangent Vectors at the end points R_1 & R_4



Cubic Hermite Spline Curve

It has Local Control over the Curve.

Let $Q(t)$ is the Curve where $t \in [0,1]$

$Q(t) = (x(t) \; y(t) \; z(t))$, $t \in [0,1)$ where all points Satisfy Cubic parametricity.

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As we know the general Curve Equation

$$P(t) = at^3 + bt^2 + ct + d \quad 0 \le t \le 1$$ t is parameter

So $x(t) = a_x t^3 + b_x t^2 + c_x t + d_x \cdot 1$

$y(t) = a_y t^3 + b_y \cdot t^2 + c_y \cdot t + d_y \cdot 1$

$z(t) = a_z t^3 + b_z \cdot t^2 \quad c_z t + d_z \cdot 1$

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \underbrace{\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix}}_{T} \cdot \underbrace{\begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \\ d_x & d_y & d_z \end{bmatrix}}_{C}$$

$$Q(t) = T \cdot C$$

but As per Rule for Specifying a spline curve we need a basis f^n matrix So

$$Q(t) = T \cdot \underbrace{M}_{\text{basis Matrix}} \cdot \underbrace{G}_{\text{Geometry Vector}} \quad \text{where } [C = M \cdot G]$$

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Let for Hermite we may write it as

$Q(t) = T \cdot M_H \cdot G_H$

M_H = Hermite basis matrix

G_H = Hermite Geometry Matrix vector

$= [t^3\ t^2\ t\ 1] \cdot M_H \cdot G_H$

$Q_x(t)\big|_{t=0} = P_1(\cancel{t}) = [0\ 0\ 0\ 1] \cdot M_H \cdot G_{H_x}$ — (A)

$Q_x(t)\big|_{t=1} = P_4(t) = [1\ 1\ 1\ 1] \cdot M_H \cdot G_{H_x}$ — (B)

$Q'_{(x)}t\big|_{t=0} = R_1(t)\ [0\ 0\ 1\ 0] \cdot M_H \cdot G_{H_x}$ — (C)

$Q'_{(x)}t\big|_{t=1} = R_4(t)\ [3\ 2\ 1\ 0]\ M_H \cdot G_{H_x}$ — (D)

From A, B, C, D Equations

$$\begin{bmatrix} P_1(x) \\ P_4(x) \\ R_1(x) \\ R_4(x) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \underbrace{M_H \cdot G_{H_x}}_{C}$$

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$$M_H \cdot G_{Hx} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} P_{1x} \\ P_{4x} \\ R_{1x} \\ R_{4x} \end{bmatrix}$$

So

$$M_H = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \;\&$$

$$G_{Hx} = \begin{bmatrix} P_{x_1} \\ P_{x_2} \\ R_{1x} \\ R_{4x} \end{bmatrix}$$

$$Q(t) = T \cdot M_H \cdot G_H$$

$$\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{pmatrix}$$

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$$Q(t) = (2t^3 - 3t^2 + 1) P_1 + (-2t^3 + 3t^2) P_4 + (t^3 - 2t^2 + t) R_1 + (t^3 - t^2) R_4$$

$$= P_1 H_0(t) + P_4 H_1(t) + R_1 H_2(t) + R_4 H_3(t)$$

$H_0(t)$, $H_1(t)$, $H_2(t)$, $H_3(t)$ Hermite blending fn.

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