

PARAMETRIC CURVE

The Parametric representation for Curves is :-

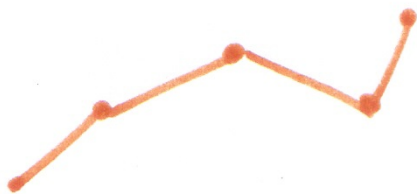
$$x = x(t)$$

$$y = y(t)$$

$$z = z(t)$$

A Curve is approximated by a piecewise polynomial Curve instead of piece linear Curve

Piecewise Linear Curve



by Polyline
& using Linear Equation

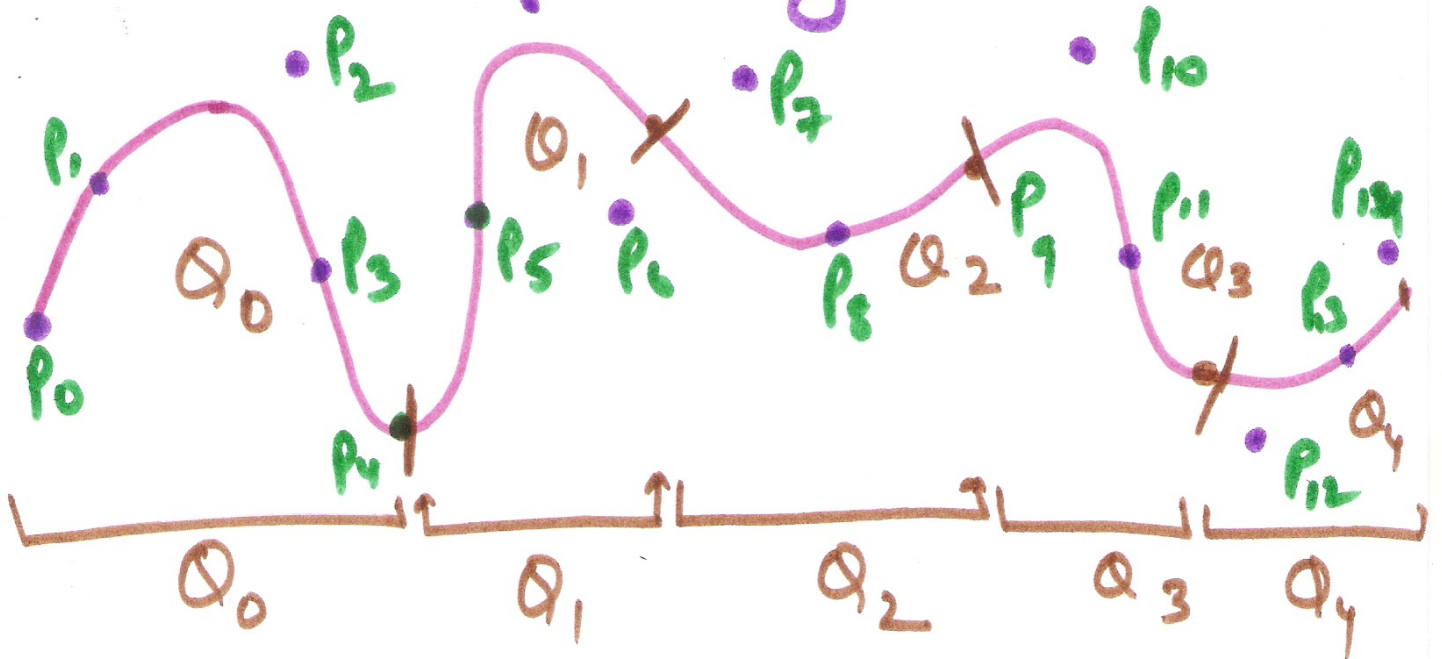
Piecewise Polynomial Curve.



Represented by Polynomial Equation.

For drawing Curve we need to specify some points through which it may or maynot completely follow:-

Let take a big Curve:-



Q_0, Q_1, Q_2, Q_3 & Q_4 are the sections or segment of big curve. & P 's are sample or control points

Each segment Q of the overall curves is given by three 3 Functions x, y, z which are cubic polynomials in the parameter t or u .

Cubic means here is that the polynomial Eq. which is used to represent the Curve is has degree of 3

The Cubic polynomials that define a Curve Segment

$$Q(t) = [x(t) \quad y(t) \quad z(t)]$$

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

①

$$0 \leq t \leq 1$$

where $T = [t^3 \quad t^2 \quad t \quad 1]$

The Coefficient matrix is defined as

$$C = \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \\ d_x & d_y & d_z \end{bmatrix} \quad \text{--- ②}$$

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So we can Rewrite equ. ① as

$$Q(t) = [x(t) \ y(t) \ z(t)] = T \cdot C \quad \text{--- ③}$$

In General:-

In this C can be further be divided

$$C = M \cdot G$$

Where $M = [m_{ij}]_{4 \times 4}$ & $G = [g_1 \ g_2 \ g_3 \ g_4]^t$

M is a 4×4 basis matrix and G is a four element Column Vector of geometric constants, called the geometric vector.

$$\text{So } Q(t) = T \cdot M \cdot G.$$

The Curve is a weighted Sum of the elements of the geometry matrix.

The weights are each Cubic polynomials of t , and are called the blending Functions:-

$$B = T \cdot M$$

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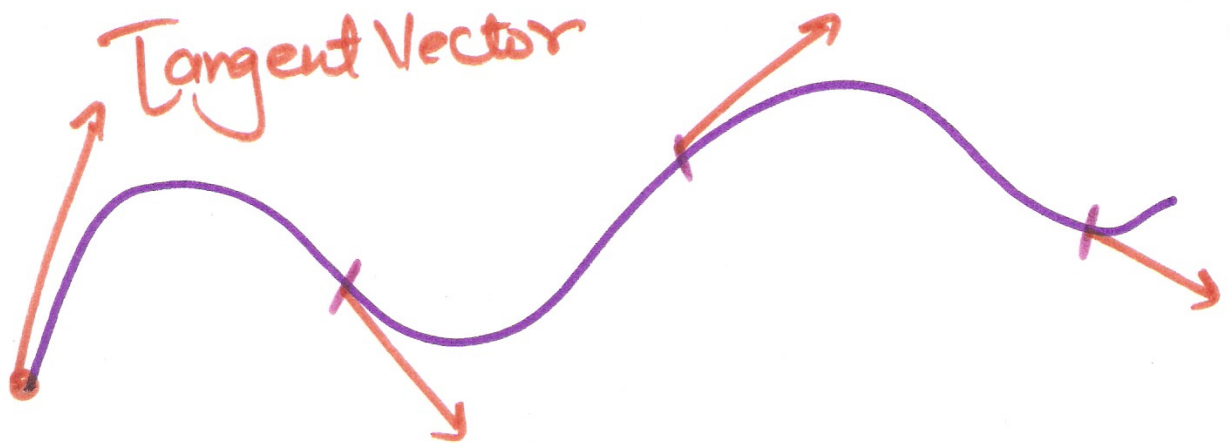
The parametric tangent-vector to the Curve is

$$\frac{d \mathbf{Q}(t)}{dt} = \mathbf{Q}'(t) = \left[\frac{dx(t)}{dt} \quad \frac{dy(t)}{dt} \quad \frac{dz(t)}{dt} \right]$$

$$= \frac{d \text{T.C}}{dt}$$

$$= [3t^2 \quad 2t \quad 1 \quad 0] \cdot C$$

$$= \left[\begin{array}{l} 3a_x t^2 + 2b_x t + c_x \\ 3a_y t^2 + 2b_y t + c_y \\ 3a_z t^3 + 2b_z t + c_z \end{array} \right]$$



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