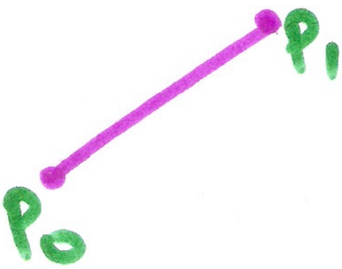
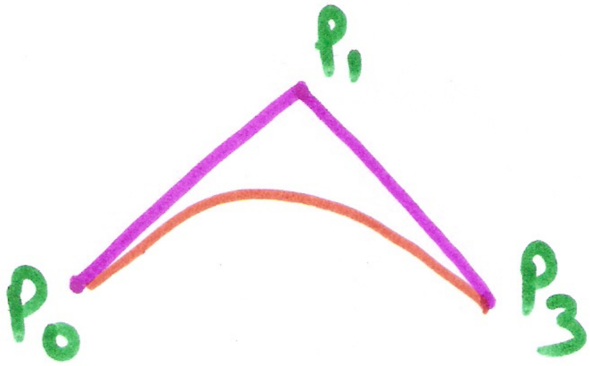
BEZIER CURVE

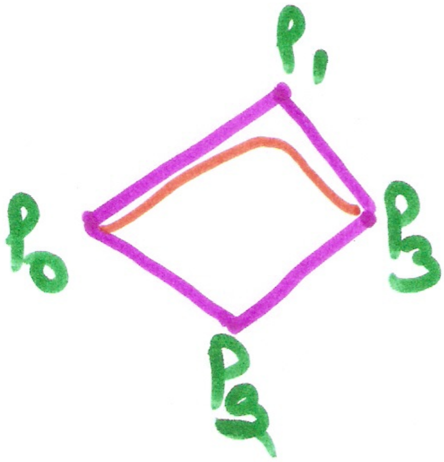
→ Bezier Curve is another approach for the construction of the Curve.

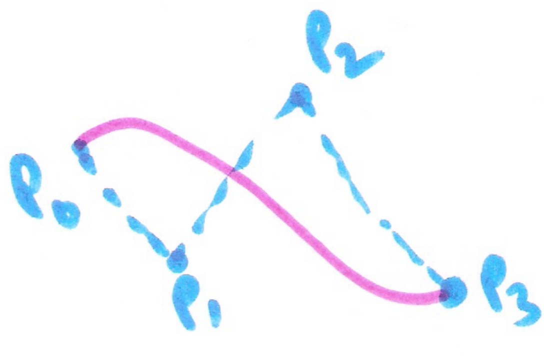
→ It is approximate spline Curve.

→ Instead of endspoints and tangents, we have four Control points in the Case of Cubic Bezier Curve.









→ Bezier splines are widely used in various CAD system, COREL DRAW Packages and many more Graphic packages.

→ As with Splines, a bezier curve can be specified with boundary conditions with a characterizing matrix or with blending f^n. For general bezier curves, the blending function specification is most convenient.

Let suppose we are given (n+1) control points positions. then $P_i = (x_i, y_i, z_i)$ with i varying from 0 to n.

These coordinate points can be blended to produce the following position vector P(u), which describes the path of an approximation so Bezier polynomial fn b/w P_0 to P_n is

$$\boxed{P(u) = \sum_{i=0}^{n} P_i \, B_{i,n}(u)} \quad 0 \le u \le 1$$

P_i control points

$B_{i,n}$ or $BEZ_{i,n}$ is Bezier fn or Bernstein Polynomials.

→ The Bernstein polynomial or the Bezier fn is very important fn which will dictate the smoothness of this curve & the weight will be dictated by boundary conditions.

$$BEZ_{i,n}(u) = {}^{n}C_{i} \cdot u^{i} (1-u)^{n-i}$$

where

$${}^{n}C_{i} = \frac{\underline{n|}}{\underline{i|}\ \underline{(n-i)|}} \quad \text{[Binomial Coefficient]}$$

For Individual Coordinates

$$X(u) = \sum_{i=0}^{n} x_i \, BEZ_{i,n}(u)$$

$$Y(u) = \sum_{i=0}^{n} y_i \, BEZ_{i,n}(u)$$

$$Z(u) = \sum_{i=0}^{n} z_i \, BEZ_{i,n}(u)$$

Bezier Curve For

3 points

$Q(u) = P_0 B_{0,2}(u) + P_1 B_{1,2}^{(u)} + P_2 B_{2,2}(u)$

Now Calculate $B_{0,2}$

$B_{0,2}^{(u)} = {}^{2}C_0 \, u^0 (1-u)^{2-0}$

$= \frac{2!}{0!\,2-0!} (1-u)^2 \cdot 1$

$= \frac{2 \times 1}{2!} (1-u)^2$

$= 1 \cdot (1-u)^2 \Rightarrow (1-u)^2$

Now $B_{1,2}(u)$ in Same way

$= 2(1-u)u$

$B_{2,2}^{(u)} = u^2$

Now using in main Equation

$Q(u) = (1-u)^2 P_0 + 2 \cdot (1-u) \cdot u \, P_1 + u^2 P_2$

$x(u) = (1-u)^2 x_0 + 2 \cdot (1-u) u \, x_1 + u^2 x_2$

4 Points

$Q(u) = P_0 B_{0,3}^{(u)} + P_1 B_{1,3}^{(u)} + P_2 B_{2,3}^{(u)} + P_3 B_{3,3}^{(u)}$

Now we will calculate $B_{0,3}^{(u)}$, $B_{1,3}(u)$ - - - as we have calculated & get

$B_{0,3}^{(u)} = (1-u)^3$

$B_{1,3}^{(u)} = 3u \cdot (1-u)^2$

$B_{2,3}^{(u)} = 3u^2 (1-u)$

$B_{3,3}^{(u)} = u^3$

Now Putting them in main Equation

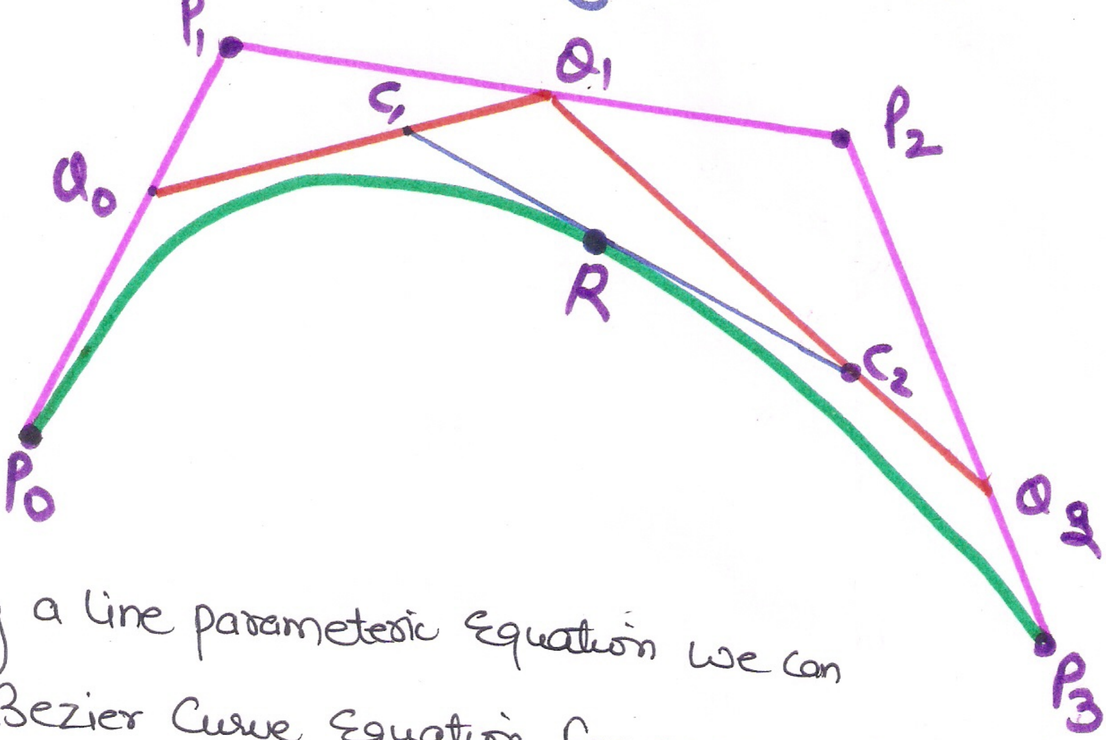
$Q(u) = P_0 (1-u)^3 + P_1 \, u (1-u)^2 + P_2 \cdot 3u^2 (1-u) + u^3 \cdot P_4$

$x(u) = (1-u)^3 x_0 + u \cdot (1-u)^2 \cdot x_1 + 3u^2 (1-u) x_2 + x_3 \cdot u^3$

$y(u) =$ In Same way

$z(u) =$

Let See the main way of Calculating the Bezier Curve or where from we get Bezier fn:-



By using a line parametric Equation we can drive Bezier Curve Equation for any no. of Control points:-

$Q_0 = (1-u) P_0 + u P_1$ [Q_0 Point on $P_0 \rightarrow P_1$

$Q_1 = (1-u) P_1 + u P_2$ Q_1 Point on $P_1 \rightarrow P_2$

$Q_2 = (1-u) P_2 + u P_3$ Q_2 Point on $P_2 \rightarrow P_3$]

$C_1 = (1-u) Q_0 + u \cdot Q_1$ [C_1 Point on $Q_0 \rightarrow Q_1$

$C_2 = (1-u) Q_1 + u \cdot Q_2$ C_2 Point on $Q_1 \rightarrow Q_2$]

$R = (1-u) C_1 + u \cdot C_2$ [R point on $C_1 \rightarrow C_2$]

Now we will use C_1, C_2, Q_0, Q_1, Q_2 Values in R:-

$$R(u) = (1-u)\,C_1 + u\cdot C_2$$

$$= (1-u)\left[(1-u)Q_0 + u\cdot Q_1\right] + u\cdot\left[(1-u)Q_1 + u\cdot Q_2\right]$$

$$= (1-u)^2 Q_0 + \underbrace{(1-u)\cdot u\cdot Q_1 + (1-u)\cdot u\cdot Q_1} + u^2\cdot Q_2$$

$$= (1-u)^2\left[(1-u)P_0 + uP_1\right] + 2(1-u)\cdot u\cdot Q_1 + u^2\left((1-u)P_2 + uP_3\right)$$

$$= (1-u)^3 P_0 + (1-u)^2\cdot u\cdot P_1 + 2(1-u)\cdot u\cdot\left[(1-u)P_1 + u\cdot P_2\right] + u^2\left((1-u)P_2 + u\cdot P_3\right)\Big]$$

$$= (1-u)^3 P_0 + \underline{(1-u)^2\cdot u\cdot P_1} + \underline{2(1-u)^2\cdot u\cdot P_1} + \underline{2(u-1)\cdot u^2 P_2} + \underline{(1-u)\cdot u^2\cdot P_2} + u^3\cdot P_3$$

$$= (1-u)^3 P_0 + 3(1-u)^2\cdot u\cdot P_1 + 3(1-u)\cdot u^2\cdot P_2 + u^3\cdot P_3$$

For x, y, z coordinate

$$(1-u)^3 x_0 + 3(1-u)^2\cdot u\cdot x_1 + 3(1-u)\cdot u^2\cdot x_2 + u^3\cdot x_3$$

Same equation which we get from Bernstein Polynomial form:—

Properties of Bezier Curves:-

(i) A very useful property of Bezier Curve is that it always passes through the first and last Control points.

$$P(0) = P_0$$
$$P(1) = P_n$$

(ii) They generally follow the shape of the Control polygon which consists of the segments joining the Control points.

(iii) The Curve is contained within Convex hull of defining polygon.

iv) The degree of the polynomial defining the Curve segment is one less than the number of defining Control polygon points. For 4 Control points the degree of polynomial is 3. i.e Cubic Bezier Curve.

(v) It is quite easy to implement.

Drawback:-

→ The degree of Bezier Curve depends on number of Control Points

2→ Bezier Curve exhibit global Control property means moving a Control point alters the shape of the whole Curve.