PARAMETRIC & GEOMETRIC CONTINUITY

A big Question Comes in mind when we join 2 piecewise polynomial parametric Curve that how to specify the smoothness of the Curve.

There are two approaches which determine the Smoothness of Curve i.e Curve Continuity:-

Parametric Continuity Conditions

C

Geometrical Continuity Conditions

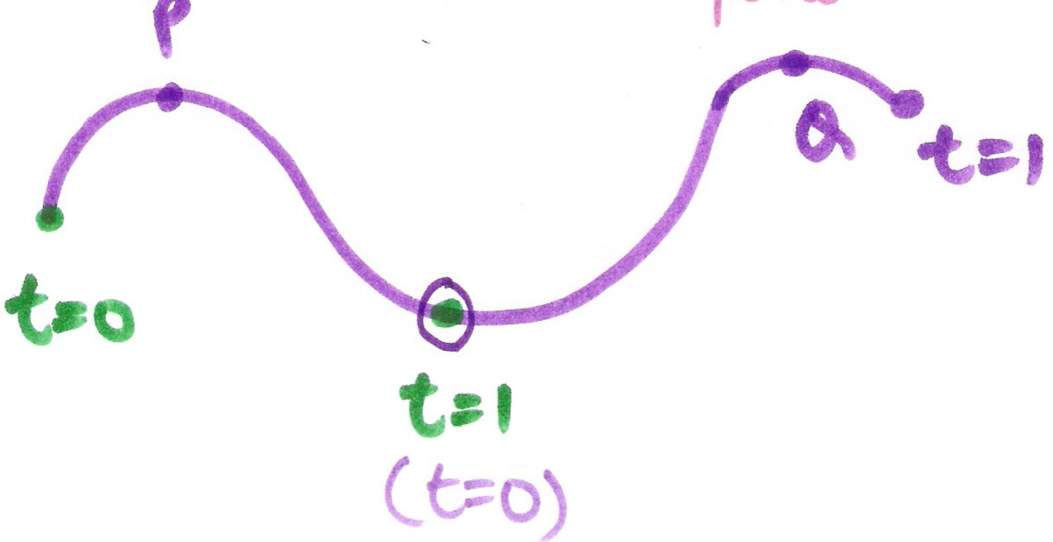
G

PARAMETRIC CONTINUITY

- Parametric Continuity deals in parametric equations associated to piecewise parametric polynomial Curve. not the Shape or appearance of the Curve.

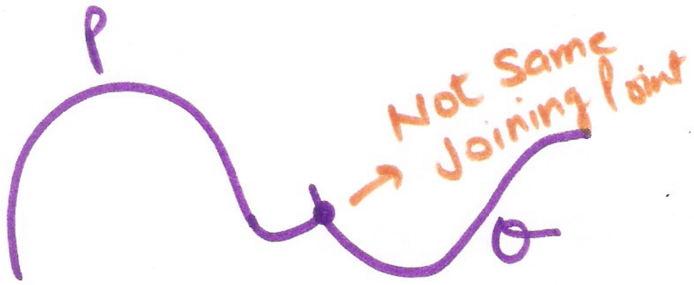
→ **Zero Order Parametric Continuity :- C^0**

C^0 Continuity means that 2 piece of Curves are joined or meet at Same point.

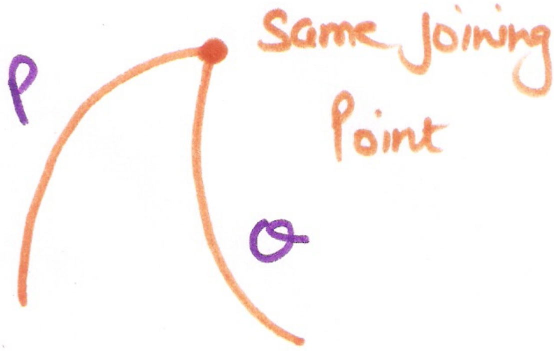


There are two piece of Curve P & Q

IF $P(t=1) = Q(t=0)$ ⇒ Zero Order Parametric Continuity.



No Zero order Parametric Cont.



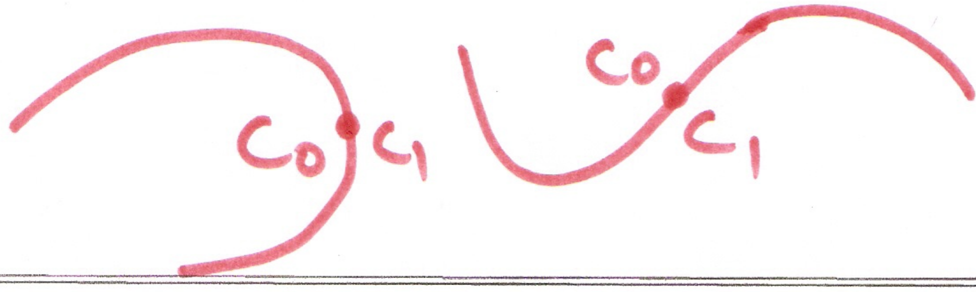
Zero order Parametric Cont

First Order Parametric Continuity (C^1):-

In First Order Parametric Continuity C' means that first parametric derivatives of the coordinate F^n for 2 successive curve sections are equal at the joining point. C1 → First Derivatives are equal.

$$P'(t=1) = Q'(t=0)$$

P' & Q' are first order Derivative.



Second Order Parametric Continuity C^2 :-

It means both First and Second derivative of 2 Curve Section are Same at the intersection point

$$P''(t=1) = Q''(t=0)$$

GEOMETRIC CONTINUITY CONDITION :-

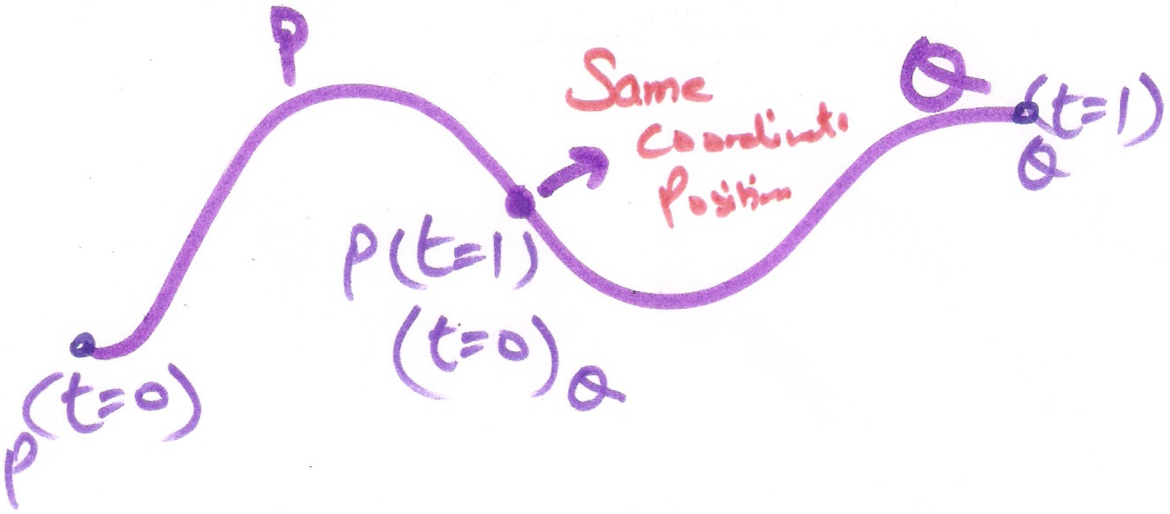
Geometric Continuity refers to the way that a Curve or Surface looks (Unit tangent or Curvature vector Continuity.)

Parametric Continuity implies geometric Continuity and Vice Versa. however exception do exist.

Zero order geometric continuity G^0

It is same as C^0 → Zero order parametric Continuity

It means two Curves Sections must have the Same Coordinate position at the boundary point.



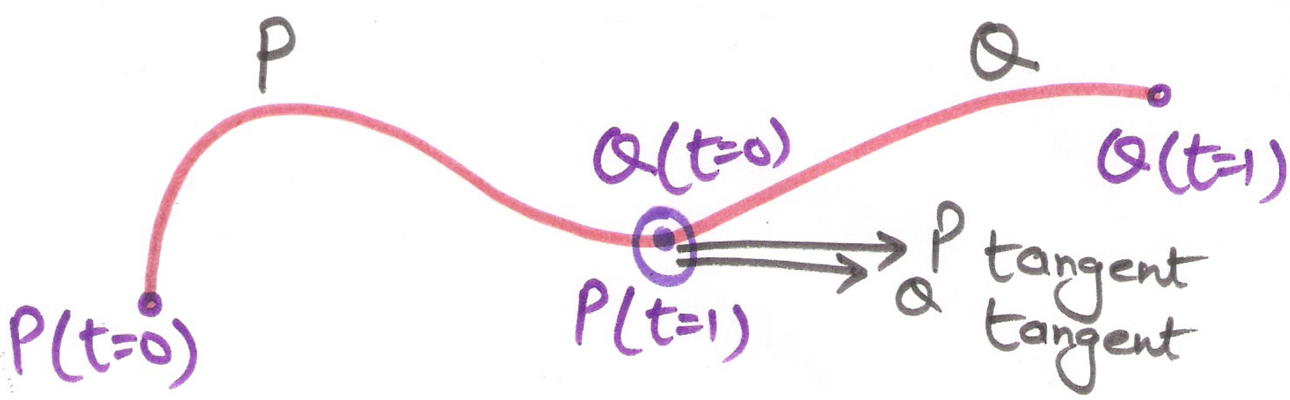
P & Q are two segments of Curves.

$$\boxed{P(t=1) = Q(t=0)}$$

First Order Geometric Continuity :- G^1

Geometric first order Continuity means that the Parametric first derivative are proportional at the intersection of 2 Successive Sections.

If P and Q are two piece of Curves, then $P'(t)_{=1}$ & $Q'(t)_{=0}$ must have Same direction of tangent Vector but not necessary the Same magnitude



Here tangent Vector has same direction but their magnitude are May or may not Same ((length)

$$\underbrace{c1'(1) = (a,b,c) \;\&\; c2'(0) = (k*a,\, k*b,\, k*c)}_{\text{Proportional.}}$$

$P'(t) \neq Q'(t)$

$G^1 \nRightarrow C^1$ May or May not

$C^1 \Rightarrow G^1$

Second Order Geometrical Continuity:-

Both first & Second derivative are propotional at their boundary point & tangent vector direction is Same & Magnitude may or may not Same.

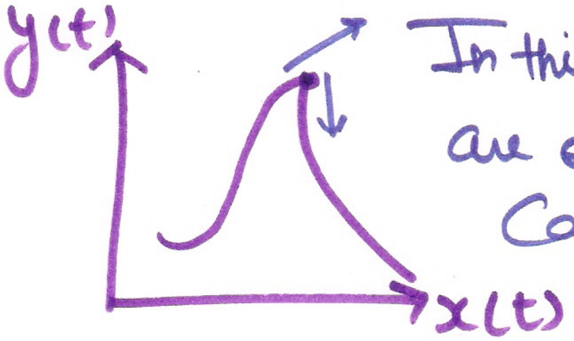
$G^2 = C^2$, $C^2 \nRightarrow G^2$

The tangent vector $Q'(t)$ is the velocity of a point on the curve with respect to parameter t.
Similarly $Q''(t)$ is the acceleration

In general C^1 continuity $\Rightarrow$ G^1 but converse is not true generally

Join point with G^1 continuity will appear just as smooth as those with C^1 continuity.

Special Case:- C^1 Continuity does not imply G^1 Continuity when segments tangent vector are $[0 \quad 0 \quad 0]$ at the join point.



In this case the tangent vectors are equal but there directions can be different